

$\theta_0 := 37 \cdot \text{deg}$ $\phi_0 := 50 \cdot \text{deg}$ Initial propagation direction expressed as polar angles.

$$k_0 := \begin{pmatrix} \cos(\phi_0) \cdot \sin(\theta_0) \\ \sin(\phi_0) \cdot \sin(\theta_0) \\ -\cos(\theta_0) \end{pmatrix} \quad k_0 = \begin{pmatrix} 0.387 \\ 0.461 \\ -0.799 \end{pmatrix} \quad \text{Initial propagation vector in model coordinates}$$

$$z := \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$k := k_0$

$z_p := k$ Define the unit vector Z in photon coordinates to be the same as the propagation vector prior to scattering

$$x_p := \frac{z \times z_p}{|z \times z_p|} \quad x_p = \begin{pmatrix} -0.766 \\ 0.643 \\ 0 \end{pmatrix} \quad \text{Define X in photon coordinates to be horizontal in model coordinates AND orthogonal to the incident ray}$$

$$x_p := \frac{1}{\sqrt{(z_{p1})^2 + (z_{p0})^2}} \cdot \begin{pmatrix} -z_{p1} \\ z_{p0} \\ 0 \end{pmatrix} \quad x_p = \begin{pmatrix} -0.766 \\ 0.643 \\ 0 \end{pmatrix} \quad \text{If denominator equals zero, then set as follows:} \quad x_p = \begin{pmatrix} -\sin(\phi_0) \\ \cos(\phi_0) \\ 0 \end{pmatrix}$$

$$y_p := z_p \times x_p \quad y_p = \begin{pmatrix} 0.513 \\ 0.612 \\ 0.602 \end{pmatrix} \quad \text{Define Y in photon coordinates to be orthogonal to the other two axes}$$

$$\begin{pmatrix} z_{p0} \\ z_{p1} \\ z_{p2} \end{pmatrix} \times \begin{pmatrix} x_{p0} \\ x_{p1} \\ 0 \end{pmatrix} = \begin{pmatrix} -z_{p2} \cdot x_{p1} \\ z_{p2} \cdot x_{p0} \\ z_{p0} \cdot x_{p1} - z_{p1} \cdot x_{p0} \end{pmatrix} \quad y_p := \begin{pmatrix} -z_{p2} \cdot x_{p1} \\ z_{p2} \cdot x_{p0} \\ z_{p0} \cdot x_{p1} - z_{p1} \cdot x_{p0} \end{pmatrix} \quad y_p = \begin{pmatrix} 0.513 \\ 0.612 \\ 0.602 \end{pmatrix}$$

$\Theta := 30 \cdot \text{deg}$ $\Phi := 135 \cdot \text{deg}$ Scattering direction relative to direction of incidence

$$k_p := \begin{pmatrix} \sin(\Theta) \cdot \cos(\Phi) \\ \sin(\Theta) \cdot \sin(\Phi) \\ \cos(\Theta) \end{pmatrix} \quad k_p = \begin{pmatrix} -0.354 \\ 0.354 \\ 0.866 \end{pmatrix} \quad \text{Scattered vector expressed in photon coordinates}$$

$$A := \text{augment}(\text{augment}(x_p, y_p), z_p)$$

$$A = \begin{pmatrix} -0.766 & 0.513 & 0.387 \\ 0.643 & 0.612 & 0.461 \\ 0 & 0.602 & -0.799 \end{pmatrix} \quad k_2 := A \cdot k_p \quad k_2 = \begin{pmatrix} 0.787 \\ 0.388 \\ -0.479 \end{pmatrix}$$

$$\begin{pmatrix} x_{p_0} & -z_{p_2} \cdot x_{p_1} & z_{p_0} \\ x_{p_1} & z_{p_2} \cdot x_{p_0} & z_{p_1} \\ 0 & z_{p_0} \cdot x_{p_1} - z_{p_1} \cdot x_{p_0} & z_{p_2} \end{pmatrix} \cdot \begin{pmatrix} k_{p_0} \\ k_{p_1} \\ k_{p_2} \end{pmatrix} = \begin{pmatrix} x_{p_0} \cdot k_{p_0} - z_{p_2} \cdot x_{p_1} \cdot k_{p_1} + z_{p_0} \cdot k_{p_2} \\ x_{p_1} \cdot k_{p_0} + z_{p_2} \cdot x_{p_0} \cdot k_{p_1} + z_{p_1} \cdot k_{p_2} \\ k_{p_1} \cdot z_{p_0} \cdot x_{p_1} - k_{p_1} \cdot z_{p_1} \cdot x_{p_0} + z_{p_2} \cdot k_{p_2} \end{pmatrix}$$

$$k := \begin{pmatrix} x_{p_0} \cdot k_{p_0} - k_2 \cdot x_{p_1} \cdot k_{p_1} + k_0 \cdot k_{p_2} \\ x_{p_1} \cdot k_{p_0} + k_2 \cdot x_{p_0} \cdot k_{p_1} + k_1 \cdot k_{p_2} \\ k_{p_1} \cdot k_0 \cdot x_{p_1} - k_{p_1} \cdot k_1 \cdot x_{p_0} + k_2 \cdot k_{p_2} \end{pmatrix} \quad k = \begin{pmatrix} 0.787 \\ 0.388 \\ -0.479 \end{pmatrix} \quad k := \frac{k}{|k|} \quad k = \begin{pmatrix} 0.787 \\ 0.388 \\ -0.479 \end{pmatrix}$$

The following is the condensed summary of how to generate a new propagation vector k, given the initial k and the scattering angles:

$$x_p := \frac{1}{\sqrt{(k_1)^2 + (k_0)^2}} \cdot \begin{pmatrix} -k_1 \\ k_0 \\ 0 \end{pmatrix} \quad x_p = \begin{pmatrix} -0.442 \\ 0.897 \\ 0 \end{pmatrix} \quad \text{If denominator equals zero, then set as follows:} \quad x_p = \begin{pmatrix} -\sin(\phi_0) \\ \cos(\phi_0) \\ 0 \end{pmatrix}$$

$\Theta := 30 \cdot \text{deg}$ $\Phi := 150 \cdot \text{deg}$ Scattering direction relative to direction of incidence

$$k_p := \begin{pmatrix} \sin(\Theta) \cdot \cos(\Phi) \\ \sin(\Theta) \cdot \sin(\Phi) \\ \cos(\Theta) \end{pmatrix} \quad k_p = \begin{pmatrix} -0.433 \\ 0.25 \\ 0.866 \end{pmatrix} \quad \text{Scattered vector expressed in photon coordinates}$$

$$k := \begin{pmatrix} x_{p_0} \cdot k_{p_0} - k_2 \cdot x_{p_1} \cdot k_{p_1} + k_0 \cdot k_{p_2} \\ x_{p_1} \cdot k_{p_0} + k_2 \cdot x_{p_0} \cdot k_{p_1} + k_1 \cdot k_{p_2} \\ k_{p_1} \cdot k_0 \cdot x_{p_1} - k_{p_1} \cdot k_1 \cdot x_{p_0} + k_2 \cdot k_{p_2} \end{pmatrix} \quad \text{Here is the new k in terms of the old k and the unit vector x.p} \quad k := \frac{k}{|k|}$$

$$k = \begin{pmatrix} 0.981 \\ 8.686 \times 10^{-4} \\ -0.195 \end{pmatrix} \quad \theta := \text{acos}[(k)_2] \quad \theta = 78.741 \text{ deg} \quad \text{Here's the new zenith angle}$$